

Inflation from Extra Dimensions

James M. Cline

Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8

The radial mode of n extra compact dimensions (the radion, b) can cause inflation in theories where the fundamental gravity scale, M , is smaller than the Planck scale M_P . For radion potentials $V(b)$ with a simple polynomial form, to get the observed density perturbations, the energy scale of $V(b)$ must greatly exceed $M \sim 1$ TeV: $V(b)^{1/4} \equiv M_V \sim 10^{-4} M_P$. This gives a large radion mass and reheat temperature $\sim 10^9$ GeV, thus avoiding the moduli problem. Such a value of M_V can be consistent with the classical treatment if the new dimensions started sufficiently small. A new possibility is that b approaches its stable value from above during inflation. The same conclusions about M_V may hold even if inflation is driven by matter fields rather than by the radion.

PACS: 98.80.Cq

McGill 99-17

The realization that compactified extra dimensions might have radii as large as 1 mm without contradicting any current experiments [1], and that this could explain why the Higgs boson mass, m_h , is much less than the Planck scale, has created a major new direction of research in particle physics. As long as the Standard Model particles are somehow stuck on our three-dimensional subspace (brane) of the $3 + n$ total spatial dimensions, the effects of the new large dimensions would have so far escaped our notice. In this picture, the Planck scale, M_P , arises as a by-product of a much smaller fundamental scale, M , and the radii of the compact dimensions, b_0 , through the relation [2]

$$M_P = 4\sqrt{\pi}M(Mb_0)^{n/2}. \quad (1)$$

If M is at the TeV scale, then the problem of why m_h is much smaller than M_P is superseded by explaining why b_0 is much larger than M^{-1} . With large enough n this is a less severe tuning problem than the original hierarchy problem.

The major new effects for phenomenology come from gravitons and their Kaluza-Klein excitations, which by $(n+4)$ -dimensional general covariance must be allowed to propagate in the extra dimensions, known as the bulk. Because their couplings to Standard Model particles are suppressed by M_P , these effects can be kept duly small, yet near the threshold for observation. In addition to the gravitons, there is a field, $b(t)$, associated with the variable size of the compact dimensions, called the radion. It can be thought of as the scale factor for the compact dimensions, where the spacetime metric has the Friedmann-Robertson-Walker-like form $g_{\mu\nu} = \text{diag}(-1, a^2, a^2, a^2, b^2, \dots, b^2)$, provided that the compact space and the brane are homogeneous and isotropic. At late times, $b(t)$ must approach its asymptotic value b_0 .

When considering inflation in such theories, it is necessary to allow for the evolution of $b(t)$ as well as the usual scale factor $a(t)$ which describes the growth of the large dimensions. In fact, the radion can economically play the role of the inflaton. Ref. [3] has explored some

of the possibilities, giving a picture in which $b \ll b_0$ at the onset of inflation, grows slowly during inflation, and attains its ultimate size only long after the end of inflation. The kind of radion potential which would give this kind of behavior is rather complicated: for $b \sim b_i \ll b_0$, where b_i is the value of b during inflation, $V(b) \sim b^{2n}$; for $b_i \ll b \ll b_0$, $V(b) \sim b^{-p}$, with $p > 0$; and for $b \gg b_0$ one expects that $V(b) \sim b^n$, since this is how the contribution from the bulk cosmological constant scales with b . The complications arise from the requiring the right magnitude of density perturbations without having to introduce any energy scales which are much greater than the new gravity scale M .

In this letter we take a different attitude; we assume that the radion potential has a relatively simple form, and then ask, what are the consequences? A major one is that, in order to obtain large enough density perturbations, $V(b)$ must be much greater than $(1 \text{ TeV})^4$ during inflation. While presenting a problem of naturalness, this condition at the same time solves the moduli problem of the radion. In this scenario the radion stabilizes immediately following inflation. There is also a new possibility: b can overshoot its stable value and approach b_0 from above during inflation. We will present numerical solutions of the evolution equations to illustrate these outcomes for specific choices of the radion potential. We also consider conventional inflation driven by matter fields after the radion has stabilized. Interestingly, a similar restriction on $V(b)$ arises as in the case when the radion is the inflaton, to avoid runaway inflation of the compact dimensions.

We start by recapitulating the equations of motion and the consistency condition for a and b , derived in [3,4]. These can be expressed in terms of Hubble parameters $H_a = \dot{a}/a$ and $H_b = \dot{b}/b$,

$$\begin{aligned} \frac{\ddot{a}}{a} + 2H_a^2 + nH_aH_b &= \frac{C_n}{b^n} \left(\left(b \frac{d}{db} - (n-2) \right) V + S_a \right) \\ \frac{\ddot{b}}{b} + (n-1)H_b^2 + 3H_aH_b &= \frac{C_n}{b^n} \left(\left(4 - 2 \frac{b}{n} \frac{d}{db} \right) V + S_b \right) \\ 3H_a^2 + \frac{1}{2}n(n-1)H_b^2 + 3nH_aH_b &= (n+2) \frac{C_n}{b^n} (V + \rho) \end{aligned} \quad (2)$$

where $C_n = (2(n+2)M^{n+2})^{-1}$, and $V(b)$ is a potential whose minimum b_0 must be consistent with eq. (1), and such that $V(b_0) \cong 0$ so that there is no cosmological constant at the end of inflation. If there is significant pressure and energy density from matter fields, say a scalar ϕ with potential V_ϕ , then $S_a = \rho + (n-1)p$, $S_b = \rho - 3p$, $\rho = \frac{1}{2}\dot{\phi}^2 + V_\phi$ and $p = \frac{1}{2}\dot{\phi}^2 - V_\phi$, with the equation of motion $\ddot{\phi} + 3H_a\dot{\phi} + V'_\phi = 0$. We shall at first assume that $S_{a,b}$ are negligible compared to $V(b)$, however.

To get inflation of a from V , it is necessary that b starts out away from its minimum, $b_i \neq b_0$, and rolls slowly toward b_0 . It is clear from eqs. (2) that the slow-roll condition is $(4 - 2\frac{b}{n}\frac{d}{db})V \ll (b\frac{d}{db} - n + 2)V$ [3]. This is only satisfied for a range of b if V has the leading behavior $V(b) \sim b^{2n}$ during inflation. For this work we will consider a potential of the form

$$V(b) = M_V^4 \left(\hat{b}^\alpha - \left(\frac{\alpha - \gamma}{\beta - \gamma} \right) \hat{b}^\beta + \left(\frac{\alpha - \beta}{\beta - \gamma} \right) \hat{b}^\gamma \right) \quad (3)$$

where $\hat{b} = b/b_0$. It has the required properties that $V = V' = 0$ at $b = b_0$; it is thus the simplest viable form. There are two possibilities for getting inflation: either $b_i < b_0$, and $\alpha = 2n < \beta < \gamma$, or $b_i > b_0$ and $\alpha = 2n > \beta > \gamma$. The shape of V in the two cases is illustrated in figure 1. The initial condition where $b_i < b_0$ was investigated in ref. [3]; here we will consider both possibilities. Notice that in either case, the middle term in V is what drives b during inflation, since $(2 - \frac{b}{n}\frac{d}{db})$ must annihilate the dominant b^{2n} term in V in order for b to roll slowly.

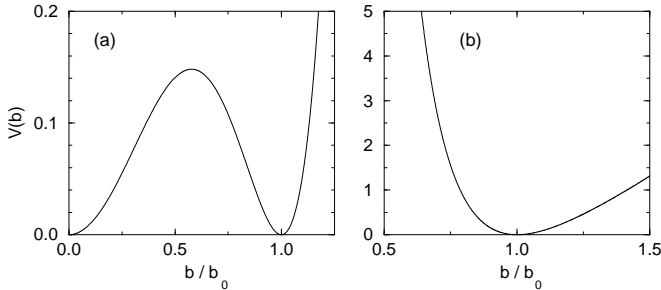


Figure 1: radion potential for the case when (a) $b_i < b_0$, so $V \sim b^{2n}$ at small b , and (b) $b_i > b_0$, so $V \sim b^{2n}$ at large b .

To numerically investigate the evolution of the scale factors and the density perturbations produced during inflation, and also for analytical insight, it is sometimes helpful to go to a dimensionless, conformal-like time variable τ , defined by $db/dt = f(b) db/d\tau$. During inflation, it is convenient to choose $f^2 = C_n b^{-n} (b\frac{d}{db} - n + 2)V \approx (n+2)C_n b^{-n}V$, while letting f become constant at the end of inflation. Then the scale factor has the simple time dependence $a(t) \cong e^{\tau/\sqrt{3}}$ during inflation.

The amount of inflation is determined mainly by b_i/b_0 , the ratio of b 's initial and final values. We will first discuss the case where $b_i < b_0$. By approximately solving

the radion equation of motion when $\ddot{b} \ll H_b \ll H_a$, one finds the conformal time dependence of b to be

$$\tau = C_b \left((b_0/b_i)^{\beta-2n} - (b_0/b)^{\beta-2n} \right) \quad (4)$$

with $C_b = [\sqrt{3}n(n+2)(\gamma - \beta)]/[2(\gamma - 2n)(\beta - 2n)^2]$. Thus the duration of inflation in conformal time goes like $(b_0/b_i)^{\beta-2n}$. Typically the conservative minimum of ~ 70 e -foldings of inflation (hence $\Delta\tau \approx 120$) can be achieved starting with modest initial conditions, *e.g.*, $b_i/b_0 \leq 0.07$, as illustrated in figure 2. But we must also insure that the observed magnitude of density perturbations is generated. At COBE (Cosmic Background Explorer) scales, $\delta\rho/\rho \approx 1 \times 10^{-5}$. In the present theory, it is straightforward to show that [3]

$$\frac{\delta\rho}{\rho} = \frac{5}{12\pi\sqrt{2n(n-1)}} \frac{H_a^2}{(bM)^{n/2}MH_b}, \quad (5)$$

by relating the radion, b , to a canonically normalized inflaton field. We can derive a simple, accurate approximation for $\delta\rho/\rho$ during inflation by solving for H_a and H_b during the slow-roll regime, when $\ddot{b}/b \ll H_b^2 \ll \ddot{a}/a \cong H_a^2$. Recalling that $V(b) \sim M_V^4$, one obtains, at an epoch when $b = b_*$,

$$\delta\rho/\rho(b_*) = C_\rho (M_V/M_P)^2 (b_0/b_*)^{\beta-2n}, \quad (6)$$

where $C_\rho = [5\sqrt{n}(n+2)(\beta - \gamma)]/[3\sqrt{3(n-1)}(2n - \beta)(2n - \gamma)]$. The most natural value for M_V is the gravity scale M . If M is only 1 TeV, as desired for solving the Higgs mass hierarchy problem, the factor $(M/M_P)^2 \sim 10^{-32}$ suppresses $\delta\rho/\rho$ enormously. To compensate, one might be tempted to take the initial value of b/b_0 to be of order $b_i/b_0 \sim (100 M_V/M_P)^{2/(\beta-2n)}$.

However, very small values of b_i cause inflation to last much longer, and the perturbations with large $\delta\rho/\rho$ were produced so early that they are still beyond the present horizon. COBE perturbations, on the other hand, had a wavelength of $\lambda_d = 7 \times 10^5$ ly at the time of photon decoupling (when $T = T_d = 0.25$ eV), and were produced

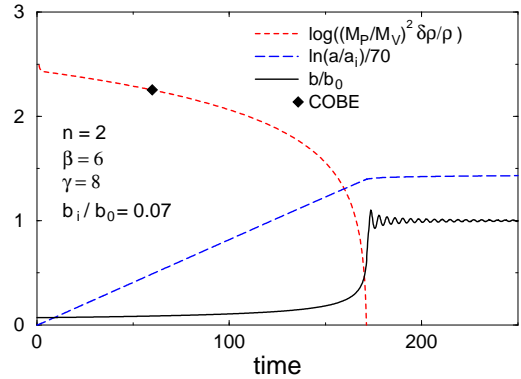


Figure 2: $\log_{10}((M_P/M_V)^2 \delta\rho/\rho)$, $\ln(a(t)/a_i)/70$ and $b(t)/b_0$, and as a function of conformal time, as described in the text. The epoch at which COBE fluctuations were produced is marked by the diamond.

N e -foldings before the end of inflation, with N given by

$$N \approx \ln \left(\lambda_d H_a \frac{T_d}{T_{rh}} \right) \sim \left(24 - \frac{14n}{\beta - 2n} \right) \ln 10. \quad (7)$$

The second equality was obtained by assuming $b_i/b_0 \sim (100 M_V/M_P)^{2/(\beta-2n)}$, $M_V = M = 1$ TeV, and a reheat temperature of 100 GeV; it shows that N is at most of order 10 under these assumptions. Thus the relevant perturbations were produced near the end of inflation. They are not enhanced by making b_i smaller; their size is determined by the value $b = b_*$ near the end of inflation, which is relatively close to b_0 .

From the above results, the only way to get a large enough $\delta\rho/\rho$ with our radion potential is to give it an energy scale $M_V \cong 3 \times 10^{15}$ GeV, which is much greater than the desired low gravity scale. It is difficult to explain why M_V should be so large if $M \sim 1$ TeV. On the other hand, there must be a large mass scale hidden in $V(b)$ in any case to obtain $b_0 \gg M^{-1}$. Moreover, a large scale for $V(b)$ solves a second pernicious problem: that of reheating [1,6]. The radion mass goes like $m_b \sim M_V^2/M_P \sim 10^{12}$ GeV, so it is no longer a modulus which oscillates practically forever without decaying, as in the case when $M_V = 1$ TeV. It decays quickly ($\Gamma_b \sim g_* m_b^3/M_P^2$) into all g_* species of lighter particles, since it couples to the trace of the stress-energy tensor.

The conventional theory of reheating then gives a reheat temperature of $T_{rh} \sim 10^{-1} \sqrt{\Gamma_b M_P} \sim 10^9$ GeV [7] from the decay of the radion condensate. (We verified that the resulting energy density is less than $V(b)$ at the end of inflation.) Although such a large T_{rh} will bring bulk gravitons into thermal equilibrium, they decay with a rate $\Gamma \sim g_* T_{rh}^3/M_P^2$, thus disappearing well before nucleosynthesis [8], and even before baryogenesis; they decay in equilibrium. Although some gravitons are produced with lower energy E , hence longer lifetimes, their numbers are suppressed by the factor $(E/T_{rh})^n$ coming from the production cross section. Those with $E < 10$ TeV, which would survive to the era of nucleosynthesis and destroy light elements during their decays, are therefore diluted by a factor of $\sim 10^{5n}$ relative to photons.

Once the reheating temperature is known, eq. (7) can be evaluated again to show that the COBE fluctuations were actually produced at $N = 73 + n \ln(b_*/b_0)$ e -foldings before inflation. This implicitly determines the value of b_* since $N = (\Delta\tau - \tau_*)/\sqrt{3}$, where $\Delta\tau$ is the duration of inflation, and b depends on τ according to eq. (4). Numerically solving in the example of figure 2 shows that the COBE perturbations were produced at $\tau_* = 60$, $b_*/b_0 = 0.09$. The perturbations are rather flat in this region, with a spectral index of $n_\rho - 1 = 2d \ln(\delta\rho/\rho)/d \ln(a) = -0.015$. This is well within the present observational limits, $|n_\rho - 1| < 0.2$.

One of the difficulties of making $M_V \gg M$ is the validity of the low-energy effective action which gives the

equations of motion (2) [3]. One expects that regions of homogeneity, where the spatial dependence of the metric can be neglected, will have an extent of only M_V^{-1} . If this is much smaller than the size of the compact dimensions, then the assumption of homogeneity within the compact space is unjustified. However with the potential (3), it is possible to start with $V(b_i) \ll M_V^4$ since b_i might be much smaller than b_0 . In such a region it is natural for the spatial fluctuations in the geometry to have a longer wavelength than b_i . The condition for having $V(b_i)^{1/4} < b_i^{-1}$ can be written as

$$b_i/b_0 < (M_P/M_V)^{2/(n+2)} (M/M_P)^{2/n}. \quad (8)$$

If $M = 1$ TeV, then the least fine-tuned case of $n = 7$, assuming an upper limit of 11 dimensions, gives $b_i/b_0 < 10^{-4}$. In the spirit of chaotic inflation, this is not unnaturally small. As long as there are some regions in the initial universe satisfying (8), they will start to inflate, and continue for many ($\sim 10^{4(\beta-2n)}$) e -foldings. If other regions fail to inflate because of their inhomogeneities, this need not concern us. Moreover, if M is of order M_V , eq. (8) becomes $b_i/b_0 \lesssim 10^{-12/(n(n+2))}$, which needs no fine-tuning at all if $n > 3$.

The above discussion was for $b_i < b_0$. What happens if the compact space starts out larger than its stable value? As long as the exponent β is less than $2n$ in the potential (3) and $\gamma < \beta$, the story is quite similar to the case of $b_i < b_0$. One can obtain enough inflation if $b_i/b_0 \gtrsim 6$, and the estimate of the density fluctuations again requires M_V to be near the GUT scale. However the initial conditions now appear to be quite fine-tuned: the length scale provided by the potential, M_V^{-1} , is many orders of magnitude smaller than the size of the compact space, so one must wonder how the latter came to be so smooth at the beginning of inflation. In fact, it is possible to start with $b \ll b_0$, as before, but the radion picks up so much speed that it overshoots b_0 and attains a large value which marks the onset of inflation. This situation is illustrated in figure 3. The spectral index is again $n_\rho - 1 = -0.015$ in this example.

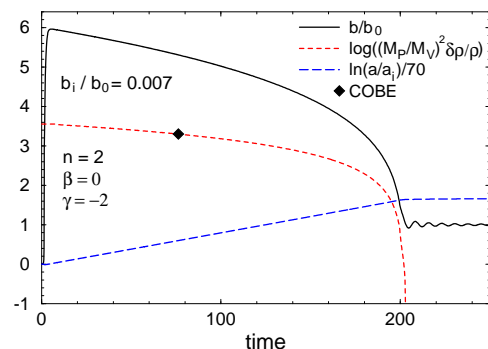


Figure 3: $b(t)/b_0$, $\log_{10}((M_P/M_V)^2 \delta\rho/\rho)$ and $\ln(a(t)/a_i)/70$ in an example where inflation occurs while $b > b_0$.

Unfortunately the most natural form for the radion potential is $V(b) \sim Ab^n - Bb^{n-2} + Cb^{n-2p}$ [4,5], which does not give inflation. For this reason it may be more convincing to drive inflation with conventional matter fields [9] or extra branes [10] after the radion has stabilized. An interesting implication of the extra dimensions is that they can be destabilized by a conventional inflaton, and begin inflating themselves. As one might expect, whether this occurs depends upon how stiff the radion potential is compared to the size of the perturbing matter potential, which we shall refer to as V_ϕ . Thus one obtains a constraint on $V(b)$ from the properties of the inflaton potential. (A different argument leading to a similar constraint was given in refs. [8] and [11].)

To see how such a constraint comes about, parametrize the radion by $b = b_0(1 + \epsilon)$ and suppose that the potential has the form $V \sim M_V^4 \epsilon^2$ in the vicinity of b_0 . Consider an inflationary phase where the pressure and energy density are dominated by the potential V_ϕ of a matter field ϕ that lives on the brane. (The same conclusions also follow if ϕ inhabits the bulk.) The source terms in eq. (2) are $S_a = (2 - n)V_\phi$ and $S_b = 4V_\phi$. A nonzero S_b will induce a shift in the radion as it tries to minimize the full source term for the b equation of motion,

$$-n^{-1}M_V^4 \epsilon (1 - (n-1)\epsilon) + V_\phi \cong 0. \quad (9)$$

If $V_\phi \ll M_V^4$ we can work to linear order in ϵ . The source term for the a equation then becomes $(n+2)C_n b^{-n} V_\phi$, in accordance with the consistency condition, third of eqs. (2). However if V_ϕ becomes too large, the quadratic term in ϵ becomes important, and it is no longer possible to satisfy eq. (9). At this point b cannot resist inflation. This situation must be avoided because, once b starts inflating, it is a runaway process; b inflates forever, as shown in figure 4.

If b remains close to b_0 , eqs. (2) reduce to the usual Friedmann equation, $H_a^2 = 8\pi\rho/3M_P^2$. In chaotic inflation with a potential of the form $V_\phi = M_\phi^{4-p}\phi^p$, it is straightforward to show that the relation between ϕ and a is $\phi^2 = \phi_i^2 - (pM_P^2/4\pi)\ln(a/a_i)$, so that initially $\phi_i \sim$ (several M_P) to get 70 e -foldings of a . The density perturbations during this epoch are given by $\delta\rho/\rho \sim (5/12\pi)H_a^2/\dot{\phi} \sim 10^3\sqrt{V_\phi}M_P^{-2} \sim 10^{-5}$. We thus obtain

$$V_\phi \sim 10^{-16}M_P^4 \lesssim M_V^4 \quad (10)$$

which is quite similar to the bound on M_V when the radion is the inflaton. In hybrid inflation models [12] this bound is softened to $M_V > 10^{-1}M_P^{3/5}m^{2/5}$.

Finally let us ask: how problematic is it for M_V to be much larger than the gravity scale, assuming it is near 1 TeV? First, it should be kept in mind that there is still no proposal for getting the required b^{2n} behavior of $V(b)$ (nor the subsequent b^{-p} behavior in the post-inflation expansion period envisioned in [3]). Whatever

mechanism that emerges to explain this might also make a large energy scale more plausible. Second, even in the most realistic model for $V(b)$, scales larger than M may be required to explain why $b_0 \gg M^{-1}$. The term in $V(b)$ coming from $\sqrt{-g}R$ in the Einstein-Hilbert action is necessarily of order $M^4(Mb)^{n-2}$, so that when $b \sim b_0$, $V \sim M^4(M_P/M)^{2-4/n}$, which is much larger than M^4 unless $n = 2$. Although not as severe a hierarchy problem as the present one, it is qualitatively similar. Third, the M_V hierarchy problem disappears altogether if we allow M to be the GUT scale rather than 1 TeV. While this value is less exciting for current accelerator experiments, it may be nature's choice, and it still presents new possibilities for the early universe.

I thank C. Burgess, N. Kaloper, G. Moore, R. Myers and M. Paranjape for helpful observations and criticisms.

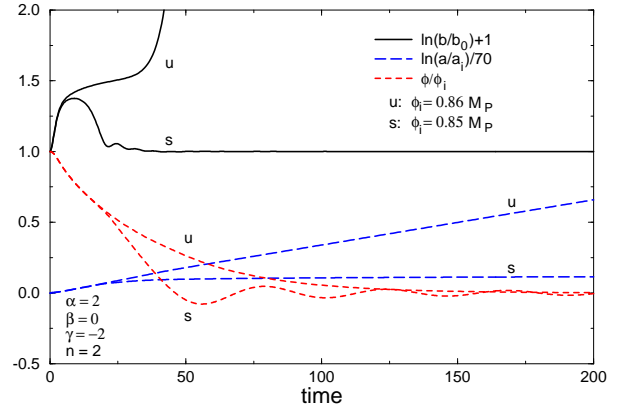


Figure 4: $\ln(b(t)/b_0) + 1$, $\ln(a(t)/a_i)/70$ and ϕ/ϕ_i for initial conditions that are stable (s) and unstable (u) against eternal inflation of b . V_ϕ is proportional to ϕ^2 in this example.

-
- [1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 086004; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.
 - [2] The factor $4\sqrt{\pi}$ insures the conventional form of the Friedmann equation (2) when $H_b = 0$ and $b = b_0$.
 - [3] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and J. March-Russell, hep-ph/9903224 (1999).
 - [4] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124 (1998)
 - [5] T. Banks, M. Dine and A. Nelson, hep-th/9903019 (1999).
 - [6] C. Csaki, M. Graesser and J. Terning, hep-ph/9903319 (1999).
 - [7] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic Publishers (1990).
 - [8] K. Benakli and S. Davidson, hep-ph/9810280 (1998).
 - [9] N. Kaloper and A. Linde, hep-th/9811141 (1998).
 - [10] G. Dvali and S.H.H. Tye, Phys. Lett. B450 (1999) 72.
 - [11] D. Lyth, Phys. Lett. B448 (1999) 191.
 - [12] A.D. Linde, Phys. Rev. D49 (1994) 748.